Math204

Homework 1.1

1) For each of the following ordinary differential equations state the order and determine whether the equation is linear or nonlinear, and indicate the independent and dependent variables.
   a) $6y''' + 2y' + 9y^2 = 2\sin x$
   b) $\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} = \tan(x+t)$
   c) $x^2y''' - 6xy' + 15y = e^x$
   d) $y[1 + (y')^2] = \ln t$

2) Give the domain of the function $y = x^{2/3}$, then give the largest interval I of definition over which $y = x^{2/3}$ is a solution of the differential equation $3xy' - 2y = 0$

3) Determine whether the indicated function is an explicit solution to the given differential equation and give at least one interval of definition for each solution.
   a) $y = 1/(4 - x^2)$, $\quad y' = 2xy^2$
   b) $\theta = 2e^{3t} - e^{2t}$, $\quad \theta'' - \theta\theta' + 3\theta = -2e^{2t}$
   c) $y = x^2 - x^{-1}$ $\quad \dfrac{d^2y}{dx^2} - \dfrac{2}{x^2}y = 0$

4) Determine whether the given relation is an implicit solution to the given differential equation. Can you find an explicit solution? . If you can then give its interval of definition.
   a) $y - \ln y = x^2 + 1$ $\quad \dfrac{dy}{dx} = \dfrac{2xy}{y-1}$
   b) $\ln\left(\dfrac{2x-1}{x-1}\right) = t$ $\quad \dfrac{dx}{dt} = (x-1)(1-2x)$

5) Show that $\varphi(x) = c_1x^{-1} + c_2x + c_3x\ ln\ x + 4x^2$ is a solution to $x^3y''' + 2x^2y'' - xy' + y = 12x^2$ for any choice of the constants c_1 and c_2. Thus $c_1x^{-1} + c_2x + c_3x\ ln\ x + 4x^2$ is a three-parameter family of solutions to the differential equation.

6) Find values of *m* so that the function $y = e^{mx}$ is a solution to $y'' - 5y' + 6y = 0$.

7) Find values of *m* so that the function $y = x^m$ is a solution to $x^2y'' - 7xy' + 15y = 0$, where $x > 0$.

8) Verify that the pair of functions

$$x = \cos 2t + \sin 2t + \tfrac{1}{5}e^t,$$
$$y = -\cos 2t - \sin 2t - \tfrac{1}{5}e^t$$

is a solution of the system of differential equations:

$$x'' = 4y + e^t$$
$$y'' = 4x - e^t$$

Homework 1.2

1) As in example 2, $y = \frac{1}{x^2+c}$ is a one-parameter family of solutions of the first-order differential equation $y' + 2xy^2 = 0$. Find a solution of the first-order IVP consisting of this differential equation and the initial condition $y(2) = \frac{1}{3}$. Give the largest interval *I* over which the solution is defined.

2) Verify that $x = c_1 e^t + c_2 e^{-2t}$ is a two-parameter family of solutions of the second-order differential equation $x'' + x' - 2x = 0$. Then find a solution of the second-order IVP consisting of this differential equation and the initial conditions $x(1) = 1, x'(1) = 0$.

3) Verify that $x = c_1 \cos t + c_2 \sin t$ is a two-parameter family of solutions of the second-order differential equation $x'' + +x = 0$. Then find a solution of the second-order IVP consisting of this differential equation and the initial conditions $x(\pi/4) = \sqrt{2}, x'(\pi/4) = 2\sqrt{2}$.

4) Determine a region of the *xy*-plane for which the given differential equation would have a unique solution whose graph passes through a point (x_o, y_o) in the region.
   a) $\frac{dy}{dx} - y = x$
   b) $(8 + y^3)y' = x^2$
   c) $x \frac{dy}{dx} = y$
   d) $(x^2 + y^2)y' = x^2$

5) Determine whether Theorem 1.2.1 implies that the given IVP has a unique solution.
   a) $\frac{dy}{dx} = x^3 - y^3$, $\quad y(0) = 8$
   b) $\frac{dy}{dx} = 7x^2 - \sqrt{4 - y^2}$, $\quad y(1) = 2$
   c) $\frac{dy}{dt} + \cos y = sin^2 t$, $\quad y(\pi) = 0$

6) Determine whether Theorem 1.2.1 guarantees that the differential equation $y' = \sqrt{y^2 - 4}$ possesses a unique solution through the given point.
   a) (2,3)
   b) (5,2)
   c) (−2,3)
   d) (1, −2)